How to do Long Division

http://www.coolmath4kids.com/math-help/division/how-do-long-division

Let's just dive right in and do one! I'm going to go really slowly and I'll show each step. After you see a few examples, it's going to start making sense!

$$68 \div 2 =$$

The first thing we do is change the way the problem is written...

$$2\overline{)68}$$

The first math step is to look at that first number of the guy we are dividing into... that 6. This is the division step!

We want to see how many times 2 will go into 6... 2 goes into 6 three times, right? So, we put that 3 right above the 6:

$$\begin{array}{r} 3 \\ 2\overline{)68} \end{array} \qquad 6 \div 2 = 3$$

Here's the second step... This is the multiplication step!

Multiply the 3 and the 2 and put the answer right under the 6:

$$\begin{array}{r} 3 \\ 2\overline{)68} \\ 6 \end{array} \qquad 3 \times 2 = 6$$

Here's the third step... This is the subtraction step!

Do the subtraction...
That's 6 - 6 = 0

$$\begin{array}{r} 3 \\ 2\overline{)68} \\ \underline{-6} \\ 0 \end{array}$$

We just finished the first chunk of steps! And it wasn't that bad!

Division, then multiplication, then subtraction.

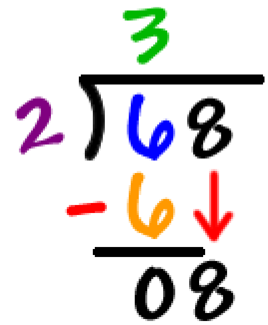
Let's call it the DMS loop!

Those letters go alphabetically!! You can use that to remember it!

OK, now we're going to do the exact same thing, but with a different number...

First thing:

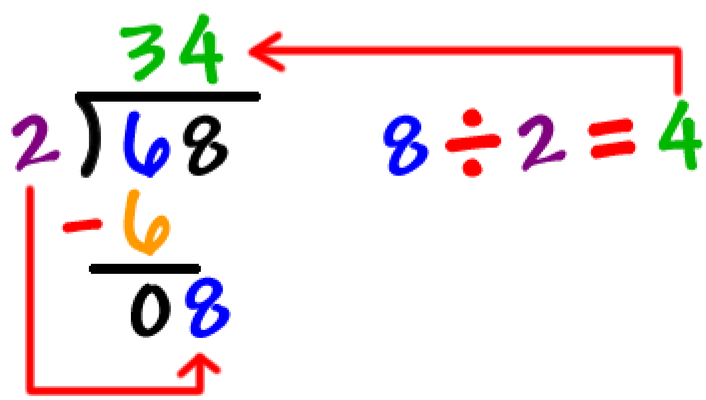
Drag the 8 down.



Now, let's go back into our division, multiplication, subtraction loop using the 8!

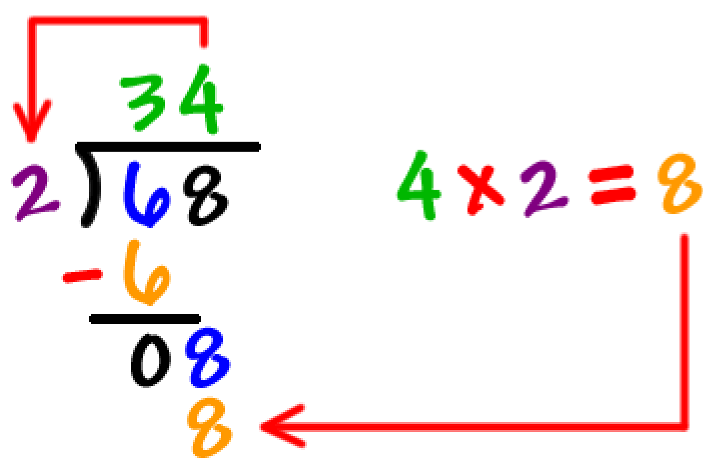
Division:

We want to see how many times 2 will go into 8... 2 goes into 8 four times... So, we put that 4 right above the 8:



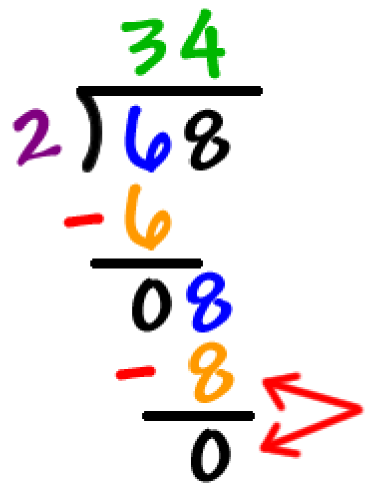
Multiplication:

Multiply the 4 and the 2 and put the answer right under the 8:



Subtraction:

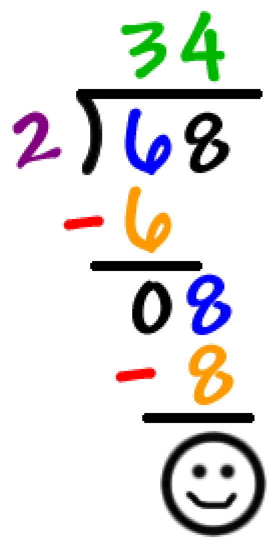
That's 8 - 8 = 0



Guess what? We are DONE!

**We used the 6 and the 8...
and we ended with a 0
at the bottom...
Which I made into a "happy
face" because I was so
happy to be done!**

The answer is 34.



So, we do our DMS loop (division-multiplication-subtraction) until we use all the numbers in the guy we are dividing into (that guy is officially called the dividend).

We have two ways to check whether our answer is right or not:

1) **Grab a calculator and do 68 divided by 2.**

2) **Use multiplication! Remember that division and multiplication go together... They undo each other! So, 34 x 2 should = 68!**

Here's another one: $75 \div 3 =$

Set it up:

$$3\overline{)75}$$

Remember, the DMS loop! Division, then multiplication, then subtraction... and we do this until we run out of numbers!

Division:

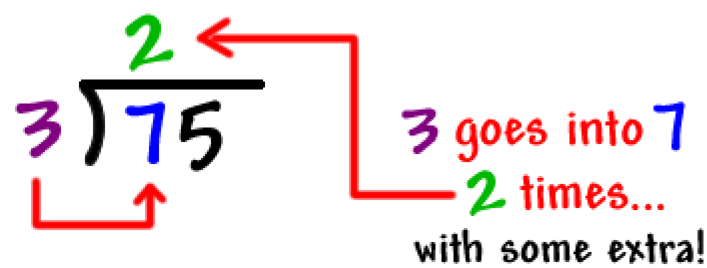
We want to see how many times 3 will go into 7...

$$3\overline{)75}$$

Hmm... It doesn't go in cleanly does it? From our times tables, we know that 3 goes into 6 and 3 goes into 9. But, 3 doesn't go into 7. That's OK.

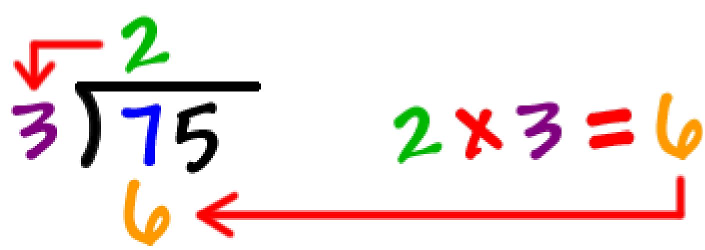
So, what does 3 go into that's a little less than 7? SIX! Three goes into 6 two times.

So, 3 goes into 7 two times... with a little left over. Put that 2 right above the 7:



Multiplication:

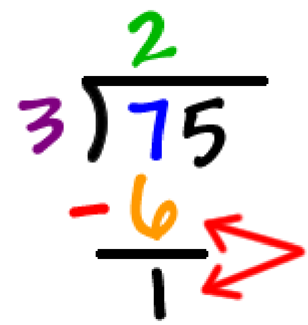
Multiply the 2 and the 3 and put the answer right under the 7:



Subtraction:

Do the subtraction...

That's 7 - 6 = 1



We just finished the first chunk of steps! And it wasn't that bad!

DMS!

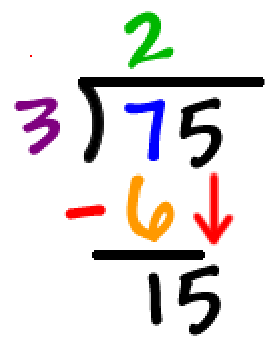
Division, multiplication, subtraction.

Now we're going to do the exact same thing, but with a different number...

The first thing is to set it up:

Drag the 5 down.

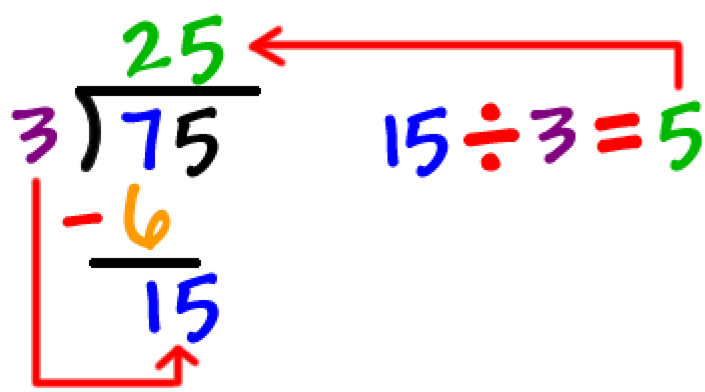
That makes it a 15...



Now, let's go back into our DMS loop using the 15!

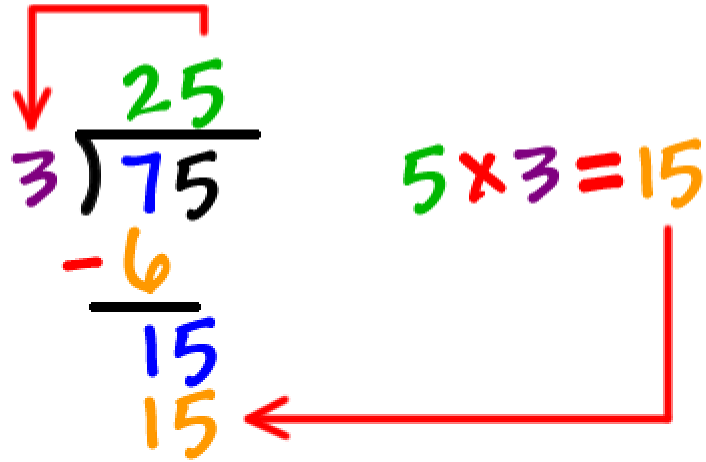
Division:

We want to see how many times 3 will go into 15... 3 goes into 15 five times... So, we put that 5 right above the 5:



Multiplication:

Multiply the 5 and the 3 and put the answer right under the 15:

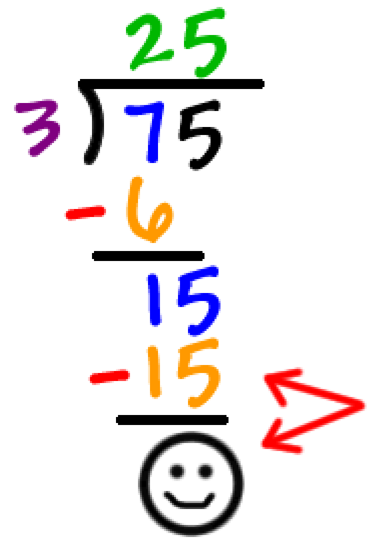


Subtraction:

That's 15 - 15 = 0

We've use all the numbers... the 7 and the 5... and we got a 0 at the end...

DONE!
The answer is 25.



Check it! 25 x 3 should = 75!

http://www.coolmath.com/algebra/05-division-of-polynomials/02-long-division-01

Long division with polynomials works the same way. You just have to deal with **x** junk along the way.

Let's walk through one:

$$(2x^2+7x-15)\div(x+5)$$

Set it up:

$$x+5\,\overline{)\,2x^2+7x-15}$$

The first thing we do is see how many times this guy goes into this guy.

So, we are looking at $2x^2 \div x$

The easiest thing to do is to write it like

$$\frac{2x^2}{x}$$ and reduce... $2x$

put it here

$$\begin{array}{r} 2x \\ x+5\,\overline{)\,2x^2+7x-15} \end{array}$$

Now, do what you did with regular long division:

multiply these

$$\begin{array}{r} 2x \\ x+5\,\overline{)\,2x^2+7x-15} \end{array}$$

Then, stick the product here and subtract.

Before we go on, I want to point out something very important to you...

$$\begin{array}{r|l} & 2x \\ \hline x+5 & 2x^2+7x-15 \\ & -(2x^2+10x) \end{array}$$

See these parentheses? If you don't put them in, you'll do the entire problem WRONG! Why?

Because that minus distributes into the second guy!

$$-(2x^2+10x)$$

Finish the subtraction!

$$\begin{array}{r|l} & 2x \\ \hline x+5 & 2x^2+7x-15 \\ & \underline{-(2x^2+10x)} \\ & -3x \end{array}$$

$$\begin{array}{r|l} & 2x \\ \hline x+5 & 2x^2+7x-15 \\ & \underline{-(2x^2+10x)} \\ & -3x \end{array}$$

Now, we see how many times this guy goes into this guy.

$$\frac{-3x}{x} = -3$$

$$\begin{array}{r|l} & 2x - 3 \\ \hline x+5 & 2x^2 + 7x - 15 \end{array}$$

$$\begin{array}{r|l} & 2x - 3 \\ \hline x+5 & 2x^2 + 7x - 15 \\ & -(2x^2 + 10x) \\ \cline{2-2} & \quad\quad -3x - 15 \end{array}$$

Bring this down.

---

① multiply

$$\begin{array}{r|l} & 2x - 3 \\ \hline x+5 & 2x^2 + 7x - 15 \\ & -(2x^2 + 10x) \\ \cline{2-2} & \quad\quad -3x - 15 \\ & \quad -(-3x - 15) \\ \cline{2-2} & \quad\quad\quad\quad\quad 0 \end{array}$$

② and subtract

When you hit 0, and there are no guys left to bring down, you're done!

So,

$$(2x^2 + 7x - 15) \div (x + 5) = 2x - 3$$

Check it by multiplying!

$$(2x-3)(x+5) = 2x^2+10x-3x-15$$
$$= 2x^2+7x-15$$

Here's another one:

$$\frac{14x^4-5x^3-11x^2-11x+8}{2x-1}$$

Set it up:

$$2x-1\overline{\big)\,14x^4-5x^3-11x^2-11x+8}$$

this into this

$$\frac{14x^4}{2x} = 7x^3$$

$$\begin{array}{r} 7x^3 \\ 2x-1\overline{\big)\,14x^4-5x^3-11x^2-11x+8} \end{array}$$

Get out a piece of paper and work along with me on this!

① multiply

$$
\begin{array}{r}
7x^3 \\
2x - 1 \overline{\big) 14x^4 - 5x^3 - 11x^2 - 11x + 8} \\
-(14x^4 - 7x^3) \\
\hline
2x^3 - 11x^2
\end{array}
$$

② subtract

③ bring him down

④ this into this

$$\frac{2x^3}{2x} = x^2$$

---

① multiply

$$
\begin{array}{r}
7x^3 + x^2 \\
2x - 1 \overline{\big) 14x^4 - 5x^3 - 11x^2 - 11x + 8} \\
-(14x^4 - 7x^3) \\
\hline
2x^3 - 11x^2 \\
-(2x^3 - x^2) \\
\hline
-10x^2 - 11x
\end{array}
$$

③

② subtract

④ this into this

$$\frac{-10x^2}{2x} = -5x$$

I'm just going to finish it off:

$$
\begin{array}{r|l}
 & 7x^3 + x^2 - 5x - 8 \\
\hline
2x - 1 & 14x^4 - 5x^3 - 11x^2 - 11x + 8 \\
 & -(14x^4 - 7x^3) \\
\hline
 & \qquad 2x^3 - 11x^2 \\
 & \quad -(2x^3 - x^2) \\
\hline
 & \qquad\quad -10x^2 - 11x \\
 & \qquad -(-10x^2 + 5x) \\
\hline
 & \qquad\qquad\quad -16x + 8 \\
 & \qquad\qquad -(-16x + 8) \\
\hline
\end{array}
$$

So,

$$\frac{14x^4 - 5x^3 - 11x^2 - 11x + 8}{2x - 1} = 7x^3 + x^2 - 5x - 8$$

Check it by multiplying (I'll wait):

$$(2x - 1)(7x^3 + x^2 - 5x - 8)$$

Things to Watch out for

Here's one thing that can happen:

Look at this:

$$(x^3 - 8) \div (x - 2) \text{ which is } x - 2\overline{)\,x^3 - 8}$$

Whenever you set these guys up, you need two things:

1 All the powers in descending order:

$$x^5 \quad x^4 \quad x^3 \quad x^2 \quad x \quad \#$$

2 All the powers represented:

For $x^3 - 8$, we have no x^2 guy

and no x guy...

So, we need to write it as

$$x^3 + 0x^2 + 0x - 8$$

We haven't changed his value... But, we've added some important place-holders. I call them "dummy guys."

If you don't have the dummy guys in the setup, you're going to get stuck with a situation that you don't know how to deal with...

Check it out:

Do the first chunk of the problem WITH the dummy guys:

$$
\begin{array}{r|l}
 & x^2 \\
\hline
x-2 & x^3+0x^2+0x-8 \\
 & -(x^3-2x^2) \\
\hline
 & \quad\;\; 2x^2
\end{array}
$$

You need an x^2 guy to stick the $-2x^2$ under!

If you didn't have the dummy guy:

$$
\begin{array}{r|l}
 & x^2 \\
\hline
x-2 & x^3-8 \\
 & -(x^3-2x^2) \\
\hline
\end{array}
$$

No like terms... Very confusing!

Try working the rest of the problem the right way. I'll do it on the next page.

$$
\begin{array}{r|l}
 & x^2+2x+4 \\
\hline
x-2 & x^3+0x^2+0x-8 \\
 & -(x^3-2x^2) \\
\hline
 & \qquad 2x^2+0x \\
 & \quad -(2x^2-4x) \\
\hline
 & \qquad\qquad 4x-8 \\
 & \qquad\quad -(4x-8) \\
\hline
\end{array}
$$

☺

So,

$$(x^3-8)\div(x-2)=x^2+2x+4$$

So far, when we've been doing these long division things, we've gotten a nice 0 at the end.

$$\overline{0}$$

This means that the divisor polynomial went into the other guy cleanly -- no remainder.

As you can imagine, things only work out nicely like this when maths teachers design them this way!

So, we're going to need to learn to deal with remainders.

Let's do a really basic review:

$$15 \div 3 = 5 \leftarrow \text{ no remainder}$$

$$15 \div 7 = 2 \quad \text{with a remainder of } 1$$

For the second one, we're really doing this:

$$\begin{array}{r|l} & \ \ 2 \\ \hline 7 & \ 15 \\ & -14 \\ \hline & \ \ \ 1 \leftarrow \text{remainder} \end{array}$$

We officially write it like this:

$$15 \div 7 = 2\frac{1}{7} \begin{array}{l} \leftarrow \text{remainder} \\ \leftarrow \text{original divisor} \end{array}$$

Well, it works the same way with polynomials.

Let's try one:

$$(x^2 - 9) \div (x + 7)$$

Set it up and remember the dummy guy:

$$x+7\,\overline{)\,x^2 + 0x - 9}$$

Start working it...

$$\begin{array}{r}
x \\
x+7\,\overline{)\,x^2 + 0x - 9} \\
-(x^2 + 7x) \\
\hline
-7x
\end{array}$$

$$\begin{array}{r}
x - 7 \\
x+7\,\overline{)\,x^2 + 0x - 9} \\
-(x^2 + 7x) \\
\hline
-7x - 9 \\
-(-7x - 49) \\
\hline
40
\end{array}$$

This does NOT go into this

$\frac{40}{x}$ does NOT reduce!

So, we stop here and the 40 is our remainder.

Here's the official answer:

$$(x^2 - 9) \div (x + 7) = x - 7 + \frac{40}{x+7}$$

original divisor

TRY IT:

$$(x^3 - 2x^2 + x - 6) \div (x - 3)$$